

# NATIONAL AIR INTELLIGENCE CENTER



ANISOPLANATISM IN ADAPTIVE OPTICS  
USING SYNTHETIC BEACONS

by

Wang Yingjian

DTIC QUALITY INSPECTED 2



Approved for public release:  
distribution unlimited

19970206 023

**HUMAN TRANSLATION**

NAIC-ID(RS)T-0494-96

8 October 1996

MICROFICHE NR:

ANISOPLANATISM IN ADAPTIVE OPTICS USING SYNTHETIC BEACONS

By: Wang Yingjian

English pages: 7

Source: High Power Laser and Particle Beams; pp. 195-198

Country of origin: China

Translated by: Leo Kanner Associates

F33657-88-D-2188

Requester: NAIC/TATD/Bruce Armstrong

Approved for public release: distribution unlimited.

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE NATIONAL AIR INTELLIGENCE CENTER.

PREPARED BY:

TRANSLATION SERVICES  
NATIONAL AIR INTELLIGENCE CENTER  
WPAFB, OHIO

#### GRAPHICS DISCLAIMER

All figures, graphics, tables, equations, etc. merged into this translation were extracted from the best quality copy available.

ANISOPLANATISM IN ADAPTIVE OPTICS  
USING SYNTHETIC BEACONS

Wang Yingjian

Anhui Institute of Optics  
and Fine Mechanics  
Chinese Academy of Sciences  
Hefei, P.O.Box 1125, 230031)

ABSTRACT

The residual phase structure function of a main laser beam with synthetic beacons is derived. The effects of focal anisoplanatism on the compensation efficiency of adaptive optical system using synthetic beacons are discussed.

Key Words: synthetic beacon, adaptive optics, focal anisoplanatism.

1. Introduction

It is well known that one of the key problems in applying an adaptive optical system in turbulence effect phase compensation for laser beams propagated through the atmosphere is the light source to be selected for the beacons. Since an appropriate star cannot always be found in the isoplanatic angle to serve as a light source for beacons, researchers place their hopes on synthetic beacons, such as fabricating light sources for synthetic beacons using laser atmospheric back-scattering or on

sodium-layers[1,2]. In doing so, a new problem arises, namely: a synthetic beacon is a "point" light source over limited distances, and the overlapping region of the synthetic beacon and the main laser beam is a cone, beyond which the phase information of the main laser beam is not carried by the beacon light. In other words, the region outside the cone forms an anisoplanatic region--focal anisoplanatism[3].

This paper first derives the residual phase structure function between the synthetic beacon and the main laser beam, and then analyzes the effects of focal anisoplanatism on the compensation efficiency of adaptive optical system using synthetic beacons.

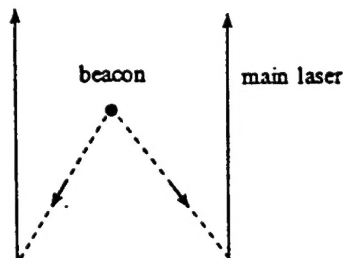


Fig. 1. Diagram of synthetic beacon

## 2. Residual Phase Structure Function

The main laser beam phase is marked with  $\Phi_m(r)$  and the beacon light phase is denoted with  $\Phi_b(r)$  as shown in Fig. 1. They can be, respectively, written as

$$\Phi_m(r) = k \int_0^H \delta n(r, z) dz \quad (1)$$

$$\Phi_b(r) = k \int_0^{H_b} \delta n \left[ \left(1 - \frac{z}{H_b}\right) r, z \right] dz \quad (2)$$

Here, let the main laser beam and the beacon light share the same wavelength;  $H_b$  in the equation is the height of the beacon light

source,  $H$  is the thickness of the atmosphere or the distance of main laser beam propagation through the atmosphere,  $k=2\pi/\lambda$  is the number of laser waves and  $\lambda$  is wavelength. Thus, the residual phase between the synthetic beacon and main laser beam is:

$$\Delta\Phi(r) = k \int_0^H \{ \delta n(r, z) - \delta n[(1 - \frac{z}{H_b})r, z] \} dz - k \int_0^H \delta n(r, z) dz \quad (3)$$

where  $\delta n$  is the fluctuation in refractivity. Under general conditions,  $H_b$  is sufficiently high so that the second term on the right-hand side of Eq. (3) can be neglected. As a result, the residual phase structure function can be derived as follows:

$$D_{\Delta\Phi}(\bar{\rho}, r) = k^2 \int_0^H dz \int_{-\infty}^{\infty} dh \{ -D_n(0, h) + D_n(\bar{\rho}, h) + 2D_n(\frac{z}{H_b}r, h) - D_n(\bar{\rho} + \frac{z}{H_b}r, h) - D_n(\bar{\rho} - \frac{z}{H_b}r, h) - D_n(\frac{h}{H_b}r, h) + \frac{1}{2} D_n[(1 - \frac{z}{H_b})\bar{\rho} - \frac{h}{H_b}r, h] + \frac{1}{2} D_n[(1 - \frac{z}{H_b})\bar{\rho} + \frac{h}{H_b}r, h] \} \quad (4)$$

where  $\bar{\rho} = r_2 - r_1$ ,  $h = z_2 - z_1$ ,  $D_n(\bar{\rho}, z_1, z_2) = c^2 \frac{(z_1 + z_2)}{2} [|\bar{\rho}|^2 + h^2]^{1/2}$  is the refractivity fluctuation structure function;  $(r_1, z_1)$ ,  $(r_2, z_2)$  are the coordinates of two points  $(r, z)$ ;  $r$  is the coordinate of laser beam cross section,  $z$  is the coordinate along the direction of laser beam propagation. When  $|\bar{\rho}| < L_0$ , according to reference [4], the integration limit of  $h$  can be extended to  $(-\infty, \infty)$ , while  $L_0$  is the turbulence external scale. Taking into consideration  $\frac{|r|}{H_b} \ll 1$ ,  $|\frac{1}{H} (1 - \frac{z}{H})r \cdot \bar{\rho}| \ll |(1 - \frac{z}{H})\bar{\rho}|$ , then

$$\begin{cases} (1 + \frac{|r|^2}{H_b^2})h^2 \approx h^2 \\ |(1 - \frac{z}{H_b})\bar{\rho} \pm \frac{h}{H_b}r|^2 + h^2 \approx |(1 - \frac{z}{H_b})\bar{\rho}|^2 + h^2 \end{cases} \quad (5)$$

From Eq. (4), the following can be obtained:

$$D_{\Delta\Phi}(\bar{\rho}, r) = 2.91k^2 \int_0^H dz c^2(z) [|\bar{\rho}|^{5/3} + 2|\frac{z}{H_b}r|^{5/3} - |\bar{\rho} + \frac{z}{H_b}r|^{5/3} - |\bar{\rho} - \frac{z}{H_b}r|^{5/3} + |(1 - \frac{z}{H_b})\bar{\rho}|^{5/3}] \quad (6)$$

Obviously, when the adaptive optical system using synthetic beacons with a limited height is used in phase compensation for laser beam atmospheric propagation, the residual phase structure function not only is a function of  $|\vec{p}| = |r_2 - r_1|$ , but also is related to the location coordinate of the laser beam cross section. Actually, it can be seen directly from Fig. 1 that the closer to the center of the laser beam, the smaller the residual phase can be, or vice versa, the larger it will be. By comparing Eq. (6) with the angular anisoplanatism in references [4] and [5], it is found that the two agree with each other in form, but the difference is that the included angle between the main laser beam and the beacon light in Eq. (6) is a function of the coordinate on the laser beam cross section  $r$ , i.e.,

$$\theta(r) = \left| \frac{r}{H_s} \right|.$$

### 3. Effect of Focal Anisoplanatism on Compensation Efficiency

The Strehl ratio is generally used to describe the compensation efficiency of an adaptive optical system. According to the definition of the Strehl ratio,

$$S_t = \left( \frac{1}{4} \pi D^2 \right)^{-1} \left\langle \left| \int u(r) w(r) dr \right|^2 \right\rangle / \int |u(r)|^2 dr \quad (7)$$

where  $D$  is aperture,  $u(r)$  is optical field function,  $w(r)$  is aperture function,  $\langle \rangle$  is the system integration average. Under general conditions,

$$w(r) = \begin{cases} 1, & |r| \leq D/2 \\ 0, & |r| > D/2 \end{cases} \quad (8)$$

$\int |u(r)|^2 dr = \pi R^2 I_0$ ,  $I_0$ , where  $I_0$  is light intensity on optical axis; for plane waves  $D=2R$ ; for the Gaussian beam  $D=2\sqrt{2}R$ . Let  $u_1(r) = u(r)/\sqrt{I_0}$ ,  $K_1(\vec{p}, r) = \frac{1}{\pi R^2} w(r - \vec{p})w(r)$ . Thus, Eq. (7) becomes

$$S_t = \left( \frac{1}{4} \pi D^2 \right)^{-1} \iint K_1(\vec{p}, r) \text{MCF}(\vec{p}, r) d\vec{p} dr \quad (9)$$

where  $MCF(\rho, r) = \langle u_1(\bar{\rho}, r) u_1^*(r) \rangle$  is the intercorrelation function of optical field; the superscript "\*" denotes composite conjugation. When the effect of amplitude fluctuation is neglected,

$$MCF(\bar{\rho}, r) = \exp\left[-\frac{1}{2} D_\phi(\bar{\rho}, r)\right] \quad (10)$$

where  $D_\phi(\bar{\rho}, r)$  is the phase structure function. When the phase fluctuation is isotropic, the phase structure function is only a function of  $|\bar{\rho}|$ . In this case, the formula for calculating the Strehl ratio is a general expression[6]

$$S_t = \left(\frac{1}{4} \pi D^2\right)^{-1} \int d\bar{\rho} k(\bar{\rho}) MCF(\bar{\rho}) \quad (11)$$

where  $K(\bar{\rho}) = \frac{1}{\pi R^2} \int dr w(r - \bar{\rho}) w(r) = \frac{2}{\pi} \left[ \arccos \frac{|\bar{\rho}|}{D} - \frac{|\bar{\rho}|}{D} \sqrt{1 - \frac{|\bar{\rho}|^2}{D^2}} \right]$ . In focal anisoplanatism, the phase structure function is a function of  $r$ . Therefore, in computing the Strehl ratio, it is necessary to adopt Eqs. (9) and (10) and carry out numerical integration. Here, we discuss only one typical case, i.e.,  $H_b > H$ . In fact, the sodium-layer synthetic beacon can satisfy this condition, because the sodium layer has a height of approximately 80-100km, while the effect of atmospheric turbulence on laser beam propagation basically concentrates in a range from 0 to 10km, or no more than 30km even if high-altitude turbulence  $H$  is taken into consideration. Moreover,  $c_n$  generally decreases with increase in altitude. Thus, Eq. (6) can be simplified as

$$D_{\Delta_\phi}(r) = 2.91 R^2 \int_0^H dz c_n^2(z) \left[ 2 \left| \frac{z}{H_b} r \right|^{5/3} \right] = 2 \left| \frac{r}{H_b} \right|^{5/3} / \theta_0^{5/3} \quad (12)$$

where  $\theta_0$  is the isoplanatic angle. In this case, it is easy to derive the computation formula for the Strehl ratio from Eqs. (10) and (9)

$$S_t = \left(\frac{1}{4} \pi D^2\right)^{-1} \int dr \exp\left[-\left| \frac{r}{H_b} \right|^{5/3} / \theta_0^{5/3}\right] \quad (13)$$

It can be seen from Eq. (13) that: (1) The larger the transmitting aperture  $D$ , the larger the anisoplanatic region can be and the poorer the compensation efficiency of the adaptive



optical system using synthetic beacons, which is just the opposite to angular anisoplanatism; (2) The larger the height of the synthetic beacon, the smaller the  $\theta(r)=|r|/H_b$ , and the better the compensation efficiency or vice versa, the poorer the compensation efficiency.

#### 4. Conclusions

The wave front residual variance can also be used to analyze the focal anisoplanatism, i.e.,

$$\begin{cases} S_i \approx \exp(-\sigma_{\Delta\phi}^2) \\ \sigma_{\Delta\phi}^2 = (\frac{1}{4} \pi D^2)^{-1} \int \langle \Delta\phi^2(r) \rangle dr \end{cases} \quad (14)$$

It is not difficult to derive a similar method as described in section 2

$$\sigma_{\Delta\phi}^2 = \frac{6}{11} \left( \frac{\theta}{\theta_0} \right)^{5/3} \quad (15)$$

where  $\theta=D/2H_b$ . As a matter of fact, it is necessary to meet the condition  $\sigma_{\Delta\phi}^2 < 1.0$  at this moment. Under similar approximation conditions, a result similar to that described above can also be obtained from Eq. (13). For instance, suppose the transmitting aperture  $D=1.0\text{m}$ ,  $H_b=100\text{km}$ ,  $\theta$  is approximately  $5\mu\text{rad}$ ; for approximately  $1.0\mu\text{m}$  laser atmospheric propagation,  $\theta$  and the atmospheric isoplanatic angle  $\theta_0$  are in the same order of magnitude, indicating that the focal anisoplanatism of synthetic beacons is a critical factor that restricts the compensation efficiency. If Rayleigh back-scattering is used as a beacon, then the problem will be even more serious. More than this,  $H_b$  is rather small at this moment, around  $20\text{km}$  or less. In such case, the atmospheric turbulence information higher than  $H_b$  also cannot be obtained from the beacon.

## REFERENCES

1. Humphreys, R. A., L. C. Bradley, and J. Hermann, "Sodium-Layer synthetic beacons for adaptive optics," The Lincoln Laboratory Journal, 1992, 5(1):45.
2. Eollars, B. G., "Atmospheric-turbulence compensation experiments using synthetic beacons," The Lincoln Laboratory Journal, 1992, 5(1):67.
3. Parenti, R. R., "Adaptive optics for astronomy," The Lincoln Laboratory Journal, 1992, 5(1):93.
4. Tatarski, B., (translated by Wen Jingsong et al.), Theory of Wave Propagation in Turbulent Atmosphere, 1st ed., Beijing Science Publishing House, 1978:133-134.
5. Tyler, G. A., "Turbulent-induced adaptive optics performance degradation evaluation in the time domain," SPIE Proc., 1983, 410:179.
6. Wang Yingjian, High Power Laser and Particle Beams, 1993, 5(4):551-556.

This paper was received for editing on January 12, 1994.  
The edited paper was received on June 29, 1994.